Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT3 **Branch: B.Tech (All)**

Semester : 1 **Date : 16/11/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) nth derivative of $y=\dfrac{1}{x+a}$ is

(A) $\dfrac{(-1)^n n!}{(x+a)^{n+1}}$ (B) $\dfrac{(-1)^{n-1} n!}{(x+a)^{n+1}}$ (C) $\dfrac{(-1)^n n!}{(x+a)^n}$ (D) none of these

b) If $y=\log(5-2x)$, then y_n equal to

(A) $\dfrac{(-1)^n n!(-2)^n}{(5-2x)^{n+1}}$ (B) $\dfrac{(-1)^n n!(-2)^n}{(5-2x)^n}$ (C) $\dfrac{(-1)^{n-1}(n-1)!(-2)^n}{(5-2x)^n}$

(D) $\dfrac{(-1)^n n!(-2)^n}{(5-2x)^{n-1}}$

c) If $1+y=e^{-x}$ or $y=e^{-x}-1$, then x equal to

(A) $-y+\dfrac{y^2}{2}-\dfrac{y^3}{3}+\dfrac{y^4}{4}+.....$ (B) $y-\dfrac{y^2}{2}+\dfrac{y^3}{3}-\dfrac{y^4}{4}+.....$

(C) $y+\dfrac{y^2}{2}+\dfrac{y^3}{3}+\dfrac{y^4}{4}+.....$ (D) none of these

d) The series $x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\dfrac{x^7}{7}+.....$ represent expansion of

(A) $\cot^{-1}x$ (B) $\tan^{-1}x$ (C) $\sin^{-1}x$ (D) $\sin x$

e) $\lim\limits_{x\to 0}\dfrac{7^x-5^x}{x}=$ _____

(A) 2 (B) $\log 2$ (C) $\log 35$ (D) $\log\left(\dfrac{7}{5}\right)$

f) $\lim\limits_{x\to\infty} x^n e^{-kx}$ (n being a positive integer and $k>0$) $=$ _____

(A) – 1 (B) 0 (C) 1 (D) None of these

g) If $Q = r\cot\theta$, then $\frac{\partial Q}{\partial \theta}$ is equal to

(A) $\cot\theta$ (B) $-r\cos ec^2\theta$ (C) $\cot\theta - r\cos ec^2\theta$ (D) $\frac{1}{2}\cot\theta$

h) If $u = f\left(\frac{x}{y}\right)$ then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ (B) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ (C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$

(D) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

i) If $u(x, y, z) = 0$ then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(A) 1 (B) -1 (C) 0 (D) none of these

j) If $x = r\cos\theta,\ y = r\sin\theta$ then $J\left(\frac{x, y}{r, \theta}\right)J'\left(\frac{r, \theta}{x, y}\right)$ is equal to

(A) 1 (B) -1 (C) 0 (D) none of these

k) The polar form of the complex number $\frac{1+i}{1-i}$ is

(A) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$ (B) $\sin\frac{\pi}{2} + i\cos\frac{\pi}{2}$ (C) $\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

(D) $\sin\frac{\pi}{4} + i\cos\frac{\pi}{4}$

l) If $z = re^{i\theta}$, then $\left|e^{iz}\right|$ is equal to

(A) $e^{r\sin\theta}$ (B) $e^{-r\sin\theta}$ (C) $e^{-r\cos\theta}$ (D) $e^{r\cos\theta}$

m) An $n\times n$ homogeneous system of equations $AX = 0$ is given. The rank of A is $r < n$. Then the system has
(A) $n - r$ independent solutions (B) r independent solutions
(C) no solution (D) n independent solutions

n) If every minor of order r of a matrix A is zero, then rank of A is
(A) greater than r (B) equal to r (C) less than or equal to r
(D) less than r

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $y = \frac{1}{x^2 + a^2}$ then find y_n. **(5)**

b) Expand $\tan^{-1} x$ up to the first four terms by Maclaurin's series. **(5)**

c) If $y = f(x + ct) + g(x - ct)$ then prove that $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$ then prove that **(5)**

$$x^2 y_{n+2} + (2n+1) x y_{n+1} + 2n^2 y_n = 0.$$

b) Prove that $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} \cdots\cdots$ **(5)**

c) Evaluate: $\lim_{x \to a} \log\left(2 - \frac{x}{a}\right) \cot(x - a)$ **(4)**

Q-4 **Attempt all questions** **(14)**

a) Evaluate: $\lim_{x \to 0} \left(\frac{1}{x}\right)^{1-\cos x}$ **(5)**

b) If $u = \frac{y^2}{x}$, $v = \frac{x^2}{y}$, evaluate $J = \left(\frac{x, y}{u, v}\right)$ and $J' = \left(\frac{u, v}{x, y}\right)$ and hence verify that $JJ' = 1$. **(5)**

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. **(4)**

Q-5 **Attempt all questions** **(14)**

a) If $u = \sec^{-1}\left(\frac{x^2 + y^2}{x - y}\right)$ then prove that **(5)**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\cot u \left(\cot^2 u + 2\right).$$

b) Evaluate: $\lim_{x \to \frac{\pi}{4}} (1 - \tan x) \sec 2x$ **(5)**

c) If $y = \cos x \cos 2x \cos 3x$ then find y_n. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Find the approximate value of $\sqrt{27} \sqrt[3]{1021}$ using partial differentiation. **(5)**

b) If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$ then prove that $\lim_{n \to \infty} x_1 x_2 x_3 \ldots x_n = -1$. **(5)**

c) Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **(4)**

Q-7 **Attempt all questions** **(14)**

a) Find the eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$. **(5)**

b) Using De Moivre's theorem prove that **(5)**

(i) $\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$

(ii) $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$

c) Prove that $\sec h^{-1}(\sin\theta) = \log \cot \frac{\theta}{2}$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ to the normal form and find its **(5)**

rank.

b) Prove that $(a+ib)^{\frac{m}{n}}+(a-ib)^{\frac{m}{n}}=2\left(a^2+b^2\right)^{\frac{m}{2n}}\cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$. **(5)**

c) Examine whether the following equations are consistent and solve them if they are consistent. **(4)**

$2x+6y+11=0,\ 6x+20y-6z+3=0,\ 6y-18z+1=0$